

# EGSDE: Unpaired Image-to-Image Translation via Energy-Guided Stochastic Differential Equations

Min Zhao, Fan Bao, Chongxuan Li and Jun Zhu

# Outline

- Score-based diffusion models (SBDM)
- Energy-guided SDE (EGSDE) and its application in (Unpaired) image-to-image (I2I)
- EGSDE in a big picture

# Denoising diffusion probabilistic models

Jonathon et al., NeurIPS 2020

Forward diffusion: a Markov chain with Gaussian kernel

$$q(\mathbf{x}^{(0)}) \longrightarrow q(\mathbf{x}^{(T)}) \approx \mathcal{N}(\mathbf{x}^{(T)}; 0, \mathbf{I})$$

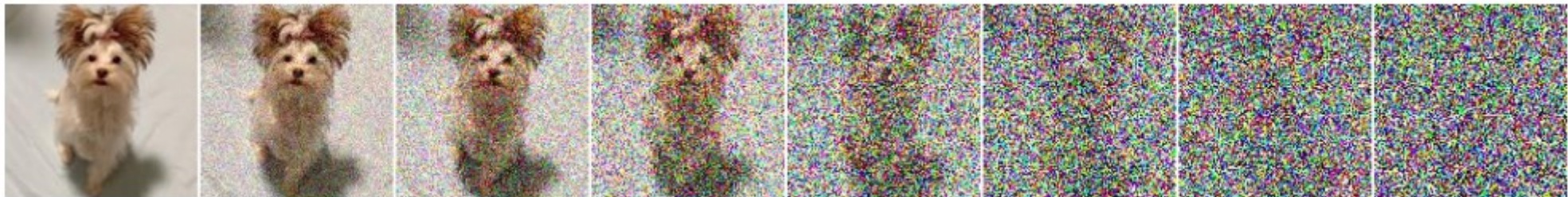
$$q(\mathbf{x}^{(t)} | \mathbf{x}^{(t-1)}) = \mathcal{N}(\mathbf{x}^{(t)}; \mathbf{x}^{(t-1)} \sqrt{1 - \beta_t}, \mathbf{I}\beta_t)$$

Decay towards origin

Add small noise

Data distribution

Gaussian noise



# Denoising diffusion probabilistic models

Jonathon et al., NeurIPS 2020

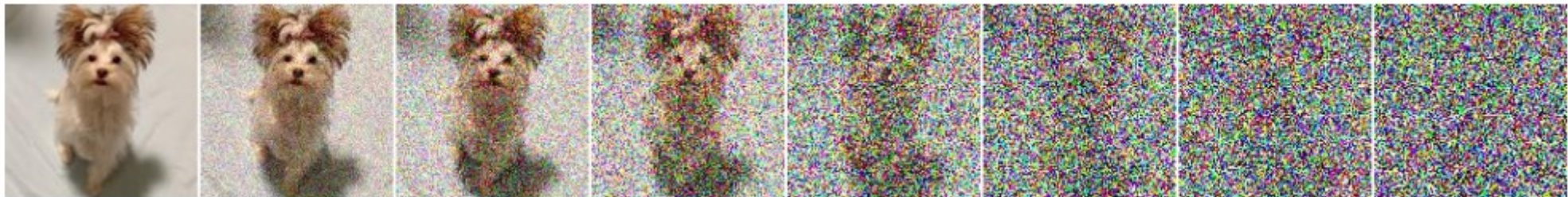
Backward diffusion: a Markov chain with Gaussian kernel

$$p(\mathbf{x}^{(0)}) \approx q(\mathbf{x}^{(0)}) \longleftarrow p(\mathbf{x}^{(T)}) = \mathcal{N}(\mathbf{x}^{(T)}; 0, \mathbf{I})$$

$$p(\mathbf{x}^{(t-1)} | \mathbf{x}^{(t)}) = \mathcal{N}(\mathbf{x}^{(t-1)}; \underbrace{f_{\mu}(\mathbf{x}^{(t)}, t)}_{\text{Learned drift and covariance functions}}, f_{\Sigma}(\mathbf{x}^{(t)}, t))$$

Data distribution

Gaussian noise



# From finite steps to infinite steps

Song et al, ICLR 2021

$$q(x_i|x_{i-1}) = \mathcal{N}(\sqrt{1 - \beta_i}x_{i-1}, \beta_i I)$$

Reparameterization

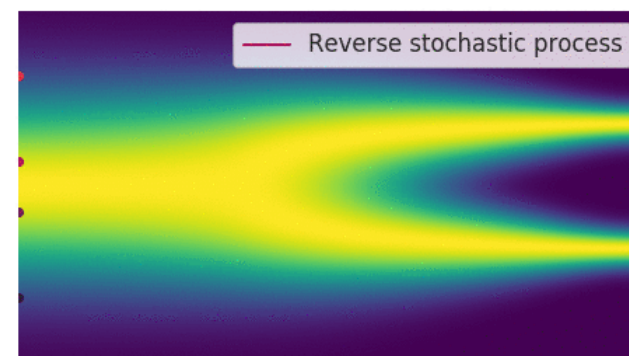
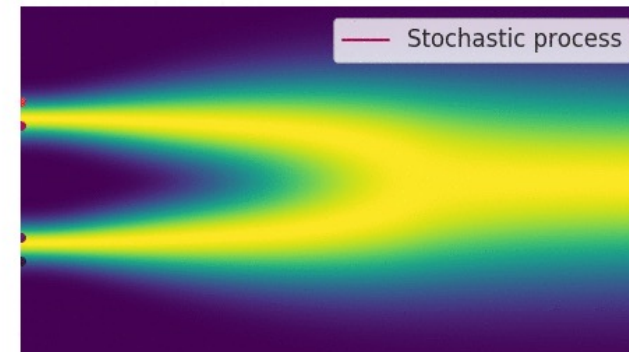
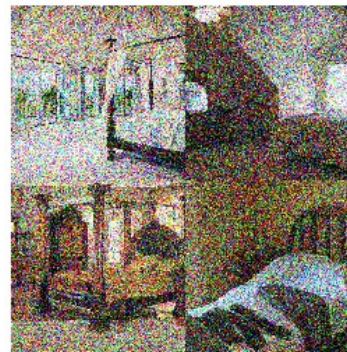
$$\mathbf{x}_i = \sqrt{1 - \beta_i}\mathbf{x}_{i-1} + \sqrt{\beta_i}\mathbf{z}_{i-1}, \quad i = 1, \dots, N, \quad \mathbf{z}_{i-1} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$$

Rescaling by  $N$  ( $\Delta t = \frac{1}{N}$ )

$$\mathbf{x}(t + \Delta t) = \sqrt{1 - \beta(t + \Delta t)\Delta t}\mathbf{x}(t) + \sqrt{\beta(t + \Delta t)\Delta t}\mathbf{z}(t)$$

Limit of  $\Delta t \rightarrow 0$

$$d\mathbf{x} = -\frac{1}{2}\beta(t)\mathbf{x} dt + \sqrt{\beta(t)} d\mathbf{w} \quad \text{VP-SDE}$$



# From finite steps to infinite steps

*Song et al, ICLR 2021*

- Forward time

$$d\mathbf{x} = \mathbf{f}(\mathbf{x}, t)dt + g(t)d\mathbf{w}$$

- Reverse time

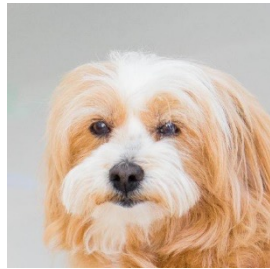
$$d\mathbf{x} = [\mathbf{f}(\mathbf{x}, t) - g(t)^2 \mathbf{s}(\mathbf{x}, t)]dt + g(t)d\bar{\mathbf{w}}$$



$$d\mathbf{x} = [\mathbf{f}(\mathbf{x}, t) - \frac{g(t)^2}{\sigma_t} \boldsymbol{\epsilon}_\theta(\mathbf{x}, t)]dt + g(t)d\bar{\mathbf{w}}$$

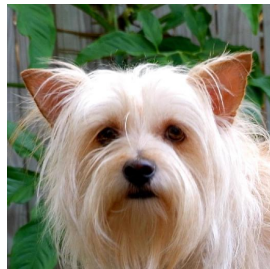
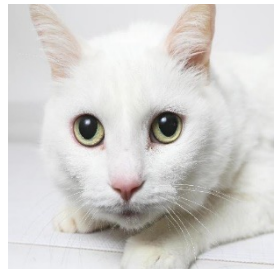
Energy-guided SDE (EGSDE) and its application in (Unpaired) image-to-image (I2I)

# Unpaired Image-to-Image Translation



⋮

⋮



source

target

Training



source image

$x_0$

$p(y_0|x_0)$



translated image

Inference



What is good translation?



source image

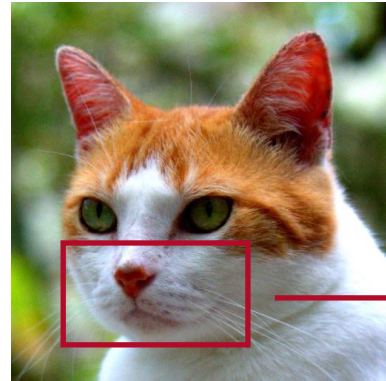
$x_0$

$p(y_0|x_0)$



translated image

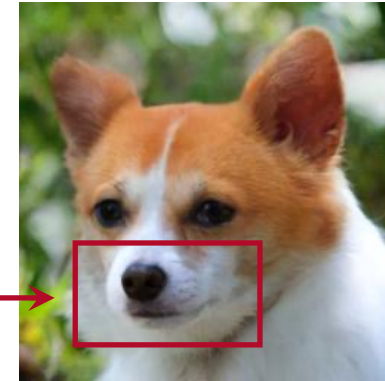
## What is good translation?



source image

$x_0$

beard, nose



translated image

- Be **realistic** for the target domain by changing the domain-specific features

## What is good translation?



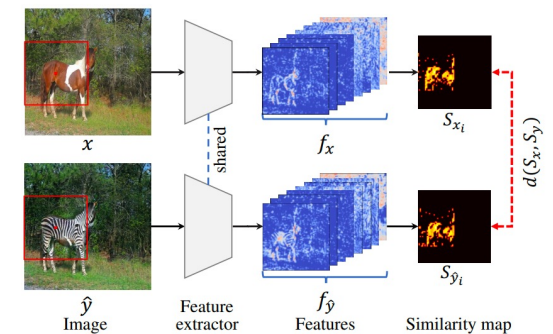
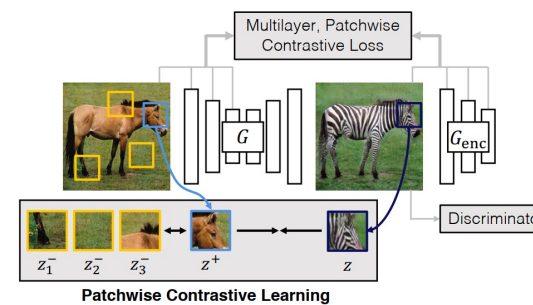
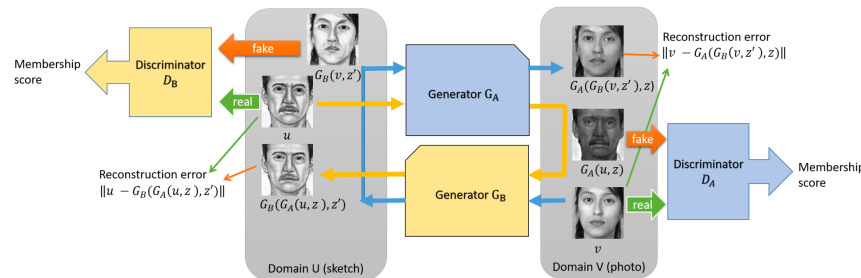
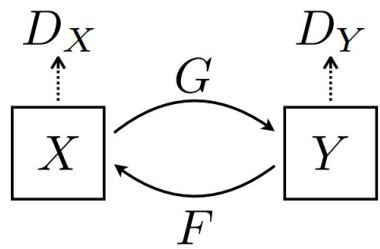
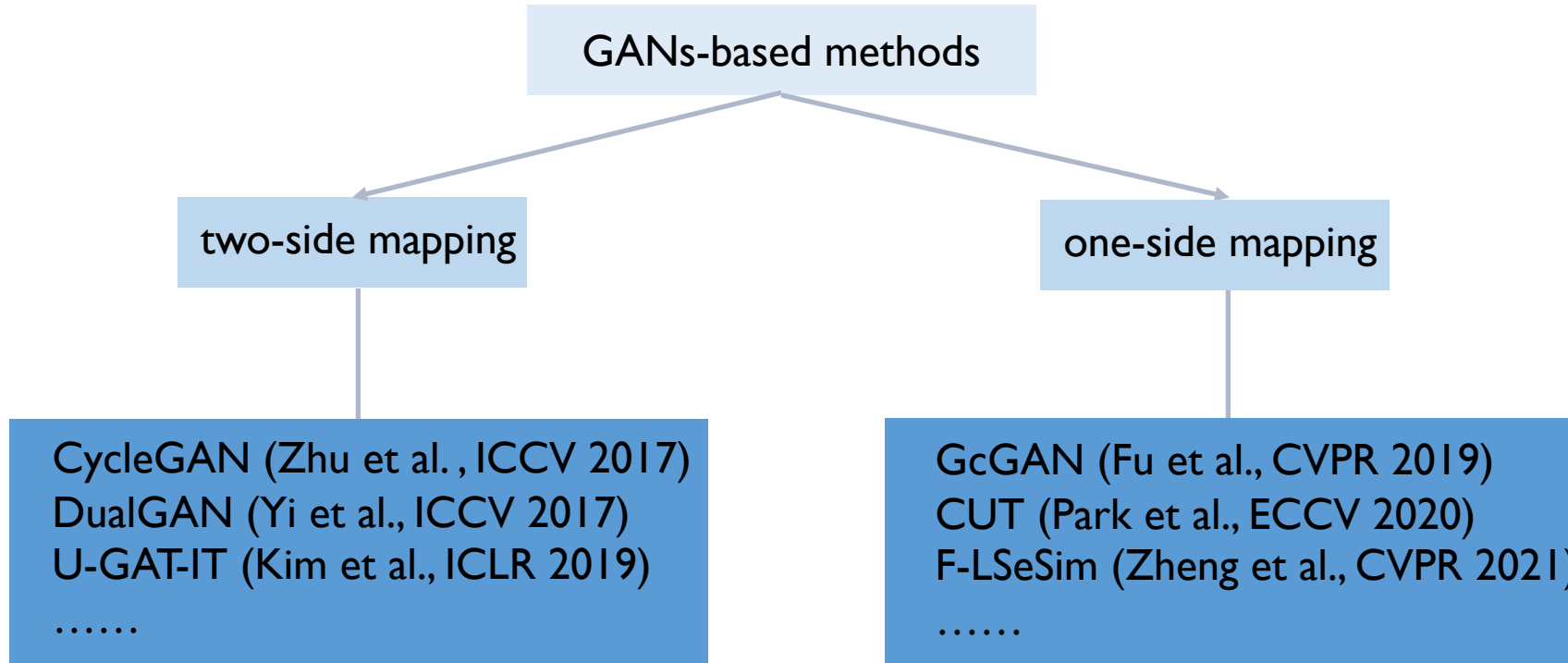
source image

$x_0$

translated image

- Be **realistic** for the target domain by changing the domain-specific features
- Be **faithful** for the source image by preserving the domain-independent features

GANs-based methods dominated this field due to their ability to generate high-quality samples



# Motivation

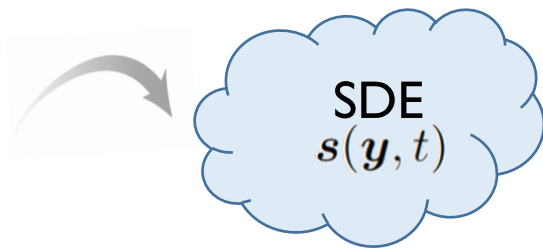
Choi et al. ICCV 2021; Meng et al, ICLR 2022;



⋮



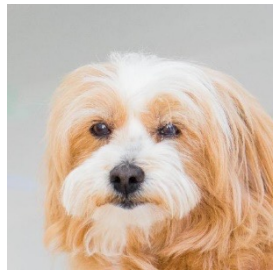
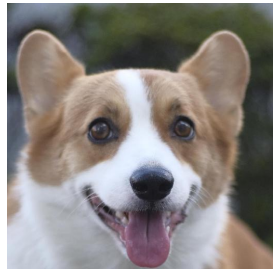
target



Training same as SDE

# Motivation

Choi et al. ICCV 2021; Meng et al, ICLR 2022;

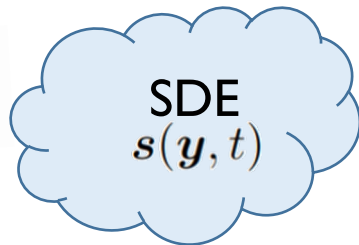


⋮



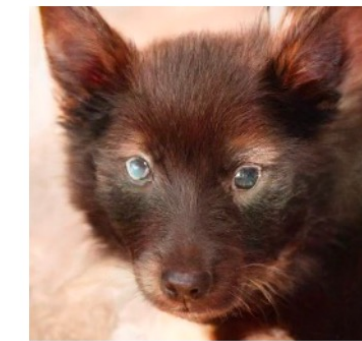
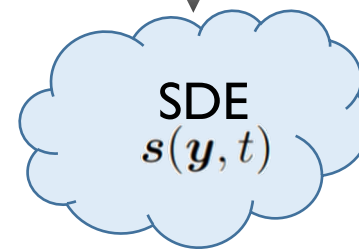
target

Training



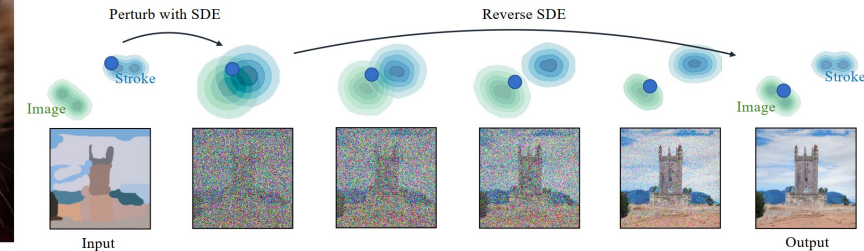
source image

$x_0$



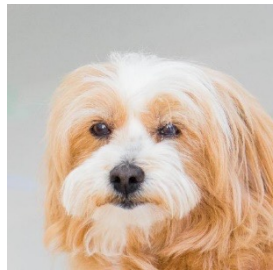
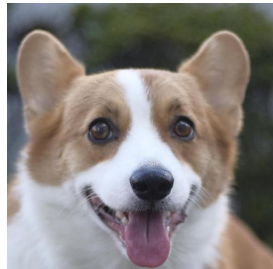
translated image

Translation only in inference



# Motivation

Choi et al. ICCV 2021; Meng et al, ICLR 2022;

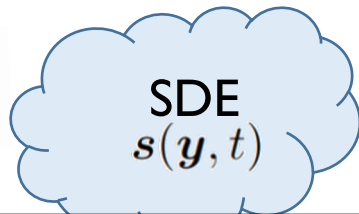


⋮



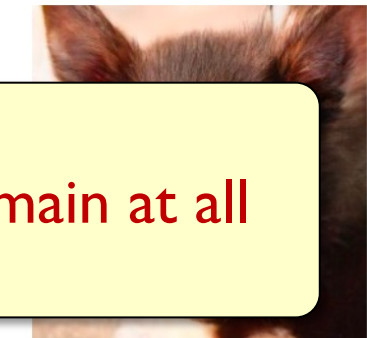
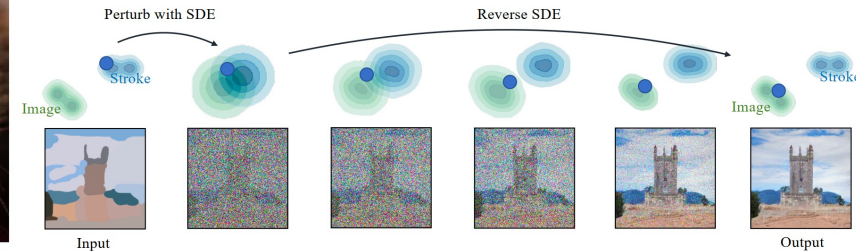
target

Training



source image

$x_0$



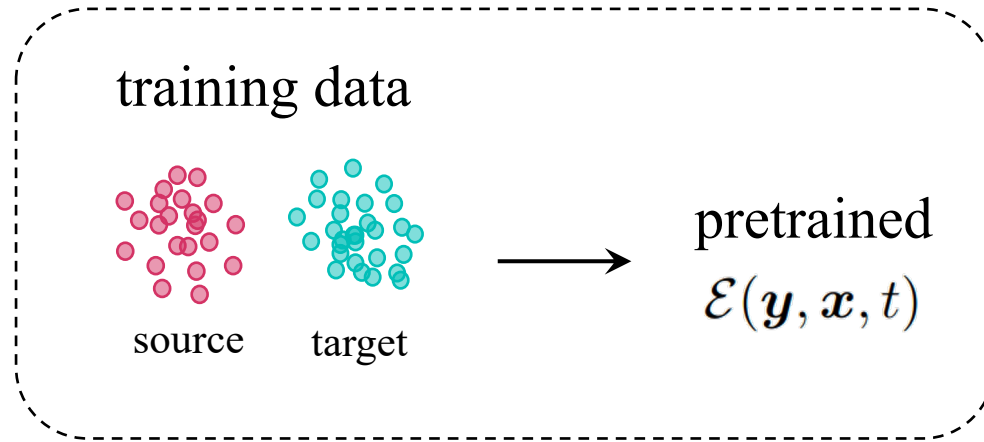
translated image

Inference

Did not leverage the training data in the source domain at all

# Energy-Guided Stochastic Differential Equations ( EGSDE )

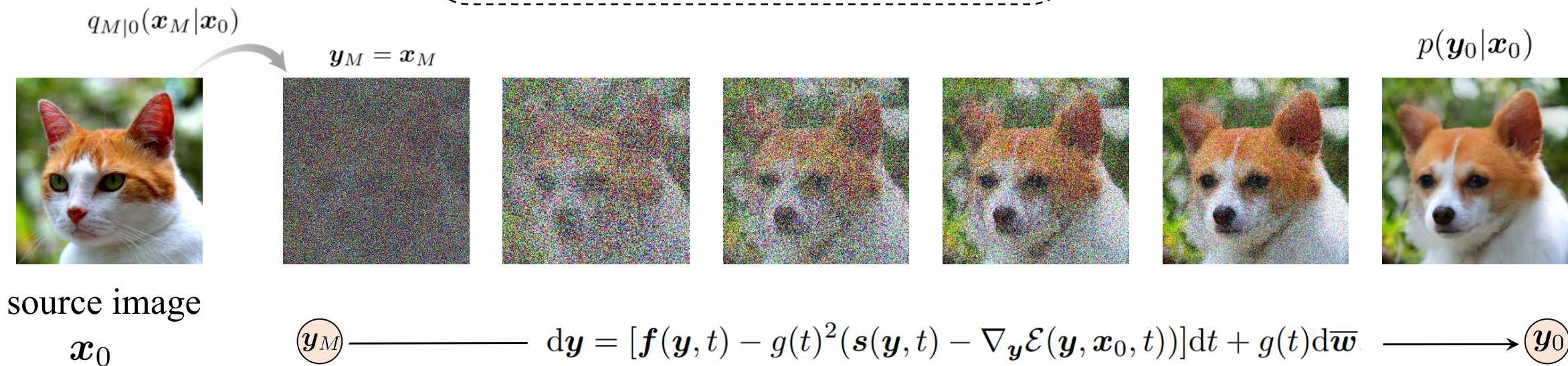
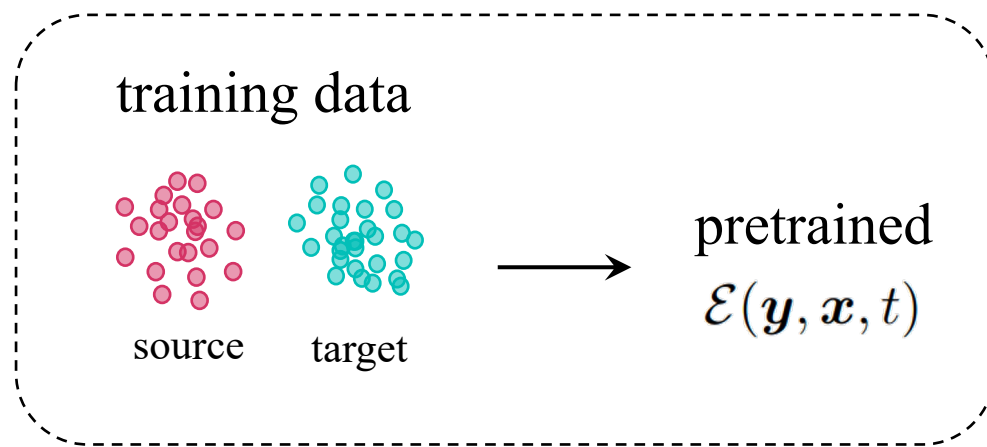
*Zhao et al, NeurIPS 2022*





# Energy-Guided Stochastic Differential Equations ( EGSDE )

Zhao et al, NeurIPS 2022



Following the SDE and decreasing the energy at the same time

## Choice of Energy

Zhao et al, NeurIPS 2022

$$\textcircled{y_M} \longrightarrow d\mathbf{y} = [\mathbf{f}(\mathbf{y}, t) - g(t)^2(\mathbf{s}(\mathbf{y}, t) - \nabla_{\mathbf{y}} \mathcal{E}(\mathbf{y}, \mathbf{x}_0, t))]dt + g(t)d\bar{\mathbf{w}} \longrightarrow \textcircled{y_0}$$

➤ Recall the goal of I2I:

Be **realistic** for the target domain by changing the domain-specific features

Be **faithful** for the source image by preserving the domain-independent features

# Choice of Energy

Zhao et al, NeurIPS 2022

$$\textcircled{y_M} \longrightarrow d\mathbf{y} = [\mathbf{f}(\mathbf{y}, t) - g(t)^2(\mathbf{s}(\mathbf{y}, t) - \nabla_{\mathbf{y}} \mathcal{E}(\mathbf{y}, \mathbf{x}_0, t))]dt + g(t)d\bar{\mathbf{w}} \longrightarrow \textcircled{y_0}$$

➤ Recall the goal of I2I:

Be **realistic** for the target domain by changing the domain-specific features

Be **faithful** for the source image by preserving the domain-independent features

➤ Decompose the energy function  $\mathcal{E}(\mathbf{y}, \mathbf{x}, t)$  as the sum of two log potential functions:

$$\mathcal{E}(\mathbf{y}, \mathbf{x}, t) = \lambda_s \mathcal{E}_s(\mathbf{y}, \mathbf{x}, t) + \lambda_i \mathcal{E}_i(\mathbf{y}, \mathbf{x}, t)$$

# Choice of Energy

Zhao et al, NeurIPS 2022

$$\textcircled{y_M} \longrightarrow d\mathbf{y} = [\mathbf{f}(\mathbf{y}, t) - g(t)^2(\mathbf{s}(\mathbf{y}, t) - \nabla_{\mathbf{y}} \mathcal{E}(\mathbf{y}, \mathbf{x}_0, t))]dt + g(t)d\bar{\mathbf{w}} \longrightarrow \textcircled{y_0}$$

➤ Recall the goal of I2I:

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➤ Decompose the energy function  $\mathcal{E}(\mathbf{y}, \mathbf{x}, t)$  as the sum of two log potential functions:

$$\begin{aligned} \mathcal{E}(\mathbf{y}, \mathbf{x}, t) &= \lambda_s \mathcal{E}_s(\mathbf{y}, \mathbf{x}, t) + \lambda_i \mathcal{E}_i(\mathbf{y}, \mathbf{x}, t) \\ &= \lambda_s \mathbb{E}_{q_{t|0}(\mathbf{x}_t|\mathbf{x})} S_s(\mathbf{y}, \mathbf{x}_t, t) - \lambda_i \mathbb{E}_{q_{t|0}(\mathbf{x}_t|\mathbf{x})} S_i(\mathbf{y}, \mathbf{x}_t, t), \end{aligned}$$

where  $q_{t|0}(\mathbf{x}_t|\mathbf{x})$  is the perturbation kernel from time 0 to time t in the forward SDE.  $S_s(\cdot, \cdot, \cdot)$  and  $S_i(\cdot, \cdot, \cdot)$  are two functions measuring the similarity between the sample and perturbed source image.

## Choice of Energy

Zhao et al, NeurIPS 2022  $\mathbf{y}_M$  —  $d\mathbf{y} = [\mathbf{f}(\mathbf{y}, t) - g(t)^2(s(\mathbf{y}, t) - \lambda_s \nabla_{\mathbf{y}} \mathcal{E}_s(\mathbf{y}, \mathbf{x}_0, t) - \lambda_i \nabla_{\mathbf{y}} \mathcal{E}_i(\mathbf{y}, \mathbf{x}_0, t))]dt + g(t)d\bar{w} \rightarrow \mathbf{y}_0$

- Suppose  $E_s(\cdot, \cdot) \in R^{C \times H \times W}$  is a **domain-specific feature extractor**,  $S_s(\cdot, \cdot, \cdot)$  is defined as the cosine similarity between the features extracted from the generated sample and the source image :

$$S_s(y, x_t, t) = \cos(E_s(y, t), E_s(x_t, t))$$

- Suppose  $E_i(\cdot, \cdot) \in R^{C \times H \times W}$  is a **domain-independent feature extractor**,  $S_i(\cdot, \cdot, \cdot)$  is defined as the negative squared L2 distance between the features extracted from the generated sample and the source image :

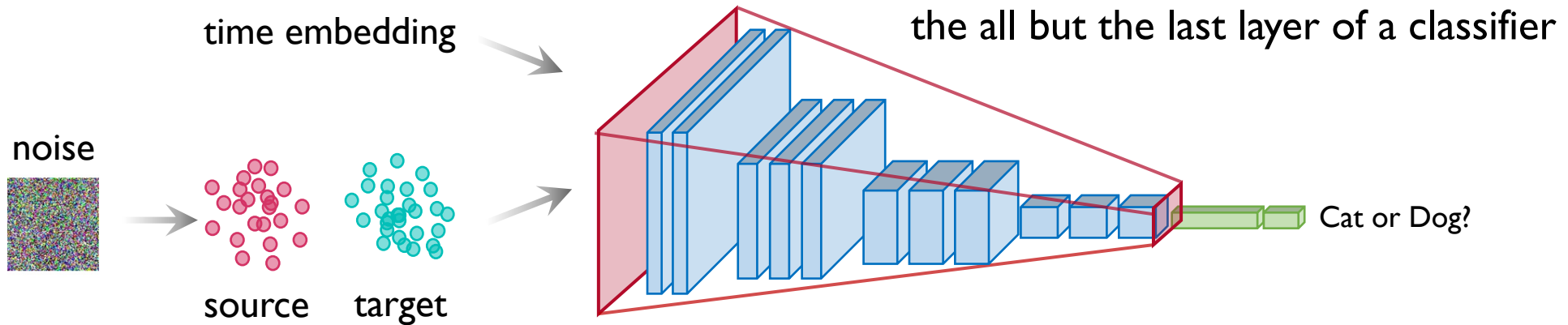
$$S_i(y, x_t, t) = -\|E_i(y, t) - E_i(x_t, t)\|_2^2$$

# Choice of Energy

Zhao et al, NeurIPS 2022

$$\mathbf{y}_M \rightarrow d\mathbf{y} = [\mathbf{f}(\mathbf{y}, t) - g(t)^2(s(\mathbf{y}, t) - \lambda_s \nabla_{\mathbf{y}} \mathcal{E}_s(\mathbf{y}, \mathbf{x}_0, t) - \lambda_i \nabla_{\mathbf{y}} \mathcal{E}_i(\mathbf{y}, \mathbf{x}_0, t))]dt + g(t)d\bar{\mathbf{w}} \rightarrow \mathbf{y}_0$$

➤ **Domain-specific feature extractor:**

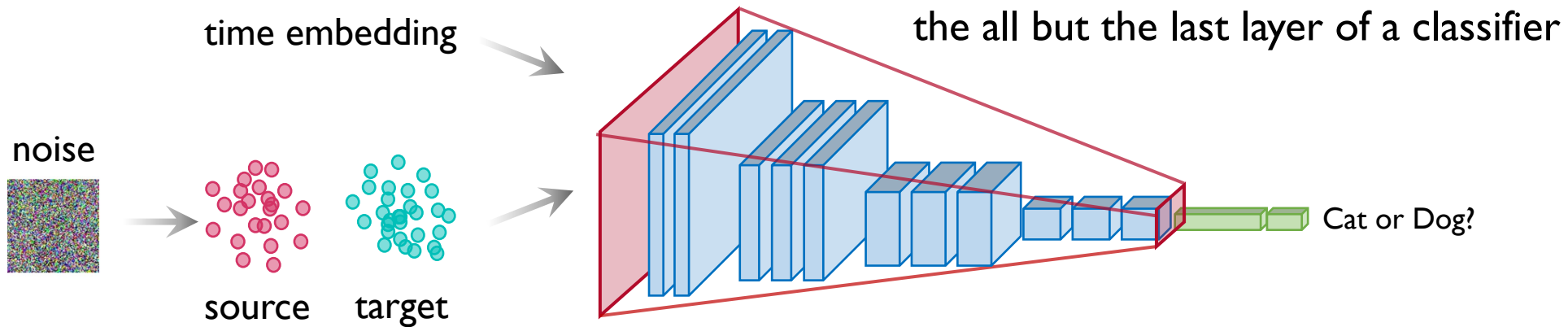


# Choice of Energy

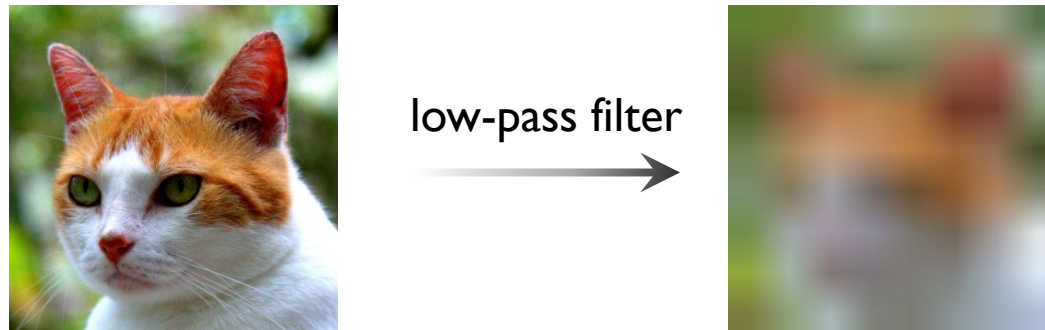
Zhao et al, NeurIPS 2022

$$\mathbf{y}_M \rightarrow d\mathbf{y} = [\mathbf{f}(\mathbf{y}, t) - g(t)^2(s(\mathbf{y}, t) - \lambda_s \nabla_{\mathbf{y}} \mathcal{E}_s(\mathbf{y}, \mathbf{x}_0, t) - \lambda_i \nabla_{\mathbf{y}} \mathcal{E}_i(\mathbf{y}, \mathbf{x}_0, t))]dt + g(t)d\bar{\mathbf{w}} \rightarrow \mathbf{y}_0$$

## ➤ Domain-specific feature extractor:

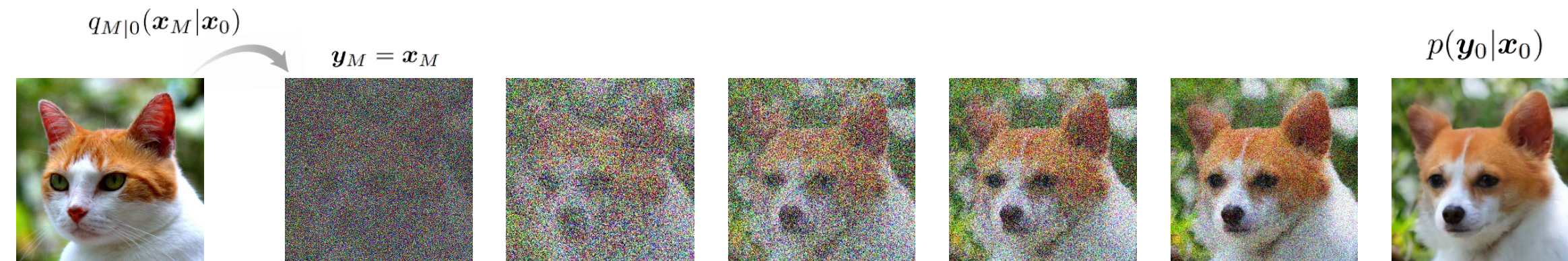
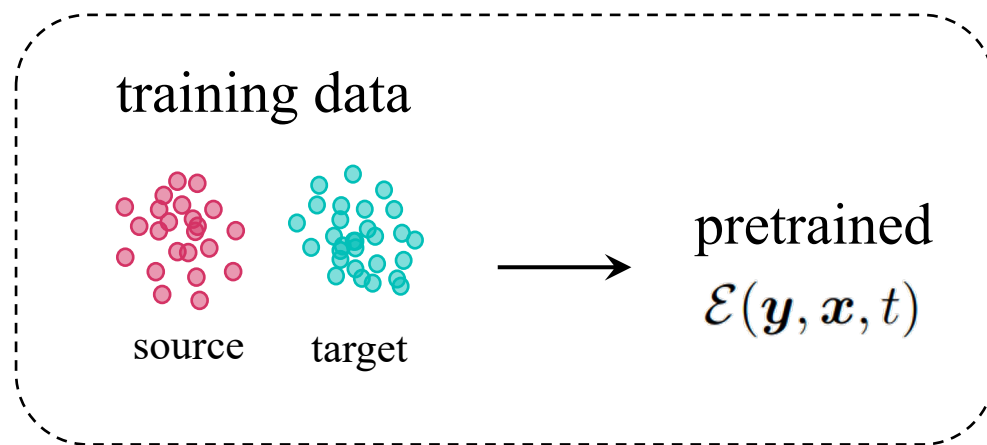


## ➤ Domain-independent feature extractor :



# Energy-Guided Stochastic Differential Equations ( EGSDE )

Zhao et al, NeurIPS 2022



source image

$\mathbf{x}_0$

$$\textcircled{\mathbf{y}_M} \longrightarrow d\mathbf{y} = [\mathbf{f}(\mathbf{y}, t) - g(t)^2 (s(\mathbf{y}, t) - \lambda_s \nabla_{\mathbf{y}} \mathcal{E}_s(\mathbf{y}, \mathbf{x}_0, t) - \lambda_i \nabla_{\mathbf{y}} \mathcal{E}_i(\mathbf{y}, \mathbf{x}_0, t))] dt + g(t) d\bar{\mathbf{w}} \longrightarrow \textcircled{\mathbf{y}_0}$$

known



# Solving the Energy-guided Reverse-time SDE

Zhao et al, NeurIPS 2022

$$\textcircled{y_M} \longrightarrow d\mathbf{y} = [\mathbf{f}(\mathbf{y}, t) - g(t)^2(s(\mathbf{y}, t) - \lambda_s \nabla_{\mathbf{y}} \mathcal{E}_s(\mathbf{y}, \mathbf{x}_0, t) - \lambda_i \nabla_{\mathbf{y}} \mathcal{E}_i(\mathbf{y}, \mathbf{x}_0, t))]dt + g(t)d\bar{\mathbf{w}} \longrightarrow \textcircled{y_0}$$

---

**Algorithm 1** EGSDE for unpaired image-to-image translation

---

**Require:** the source image  $\mathbf{x}_0$ , the initial time  $M$ , denoising steps  $N$ , weighting hyper-parameters

$\lambda_s, \lambda_i$ , the similarity function  $\mathcal{S}_s(\cdot, \cdot, \cdot), \mathcal{S}_i(\cdot, \cdot, \cdot)$ , the score function  $s(\cdot, \cdot)$

$\mathbf{y} \sim q_{M|0}(\mathbf{y}|\mathbf{x}_0)$  # the start point

$h = \frac{M}{N}$

**for**  $i = N$  to 1 **do**

$s \leftarrow ih$

$\mathbf{x} \sim q_{s|0}(\mathbf{x}|\mathbf{x}_0)$  # sample perturbed source image from the perturbation kernel

$\mathcal{E}(\mathbf{y}, \mathbf{x}, s) \leftarrow \lambda_s \mathcal{S}_s(\mathbf{y}, \mathbf{x}, s) - \lambda_i \mathcal{S}_i(\mathbf{y}, \mathbf{x}, s)$  # compute energy with one Monte Carlo

$\mathbf{y} \leftarrow \mathbf{y} - [\mathbf{f}(\mathbf{y}, s) - g(s)^2(s(\mathbf{y}, s) - \nabla_{\mathbf{y}} \mathcal{E}(\mathbf{y}, \mathbf{x}, s))]h$  # the update rule in Eq. (12)

$\mathbf{z} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$  if  $i > 1$ , else  $\mathbf{z} = \mathbf{0}$

$\mathbf{y} \leftarrow \mathbf{y} + g(s)\sqrt{h}\mathbf{z}$

**end for**

$\mathbf{y}_0 \leftarrow \mathbf{y}$

**return**  $\mathbf{y}_0$

---

Euler-Maruyama  
solver

# EGSDE as Product of Experts

Zhao et al, NeurIPS 2022

$$\tilde{p}(\mathbf{y}_t | \mathbf{x}_0) = \frac{p_{r1}(\mathbf{y}_t | \mathbf{x}_0) p_{r2}(\mathbf{y}_t | \mathbf{x}_0) p_f(\mathbf{y}_t | \mathbf{x}_0)}{Z_t} \xrightarrow{\text{Transition kernel}} \tilde{p}(\mathbf{y}_t | \mathbf{y}_s)$$

where  $p_{r1}(\mathbf{y}_t | \mathbf{x}_0)$  is the marginal distribution defined by SDE conditioned on  $\mathbf{x}_0$ ,

$$p_{r2}(\mathbf{y}_t | \mathbf{x}_0) \propto \exp(-\lambda_s \mathcal{E}_s(\mathbf{y}_t, \mathbf{x}_0, t)), p_f(\mathbf{y}_t | \mathbf{x}_0) \propto \exp(-\lambda_i \mathcal{E}_i(\mathbf{y}_t, \mathbf{x}_0, t)).$$

# EGSDE as Product of Experts

Zhao et al, NeurIPS 2022

$$\tilde{p}(\mathbf{y}_t | \mathbf{x}_0) = \frac{p_{r1}(\mathbf{y}_t | \mathbf{x}_0) p_{r2}(\mathbf{y}_t | \mathbf{x}_0) p_f(\mathbf{y}_t | \mathbf{x}_0)}{Z_t} \xrightarrow{\text{Transition kernel}} \tilde{p}(\mathbf{y}_t | \mathbf{y}_s)$$

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EGSDE

Euler-Maruyama  
solver

$$d\mathbf{y} = [\mathbf{f}(\mathbf{y}, t) - g(t)^2 (s(\mathbf{y}, t) - \lambda_s \nabla_{\mathbf{y}} \mathcal{E}_s(\mathbf{y}, \mathbf{x}_0, t) - \lambda_i \nabla_{\mathbf{y}} \mathcal{E}_i(\mathbf{y}, \mathbf{x}_0, t))] dt + g(t) d\bar{\mathbf{w}} \xrightarrow{\text{solver}} p(\mathbf{y}_t | \mathbf{y}_s)$$

# EGSDE as Product of Experts

Zhao et al, NeurIPS 2022

$$\tilde{p}(\mathbf{y}_t | \mathbf{x}_0) = \frac{p_{r1}(\mathbf{y}_t | \mathbf{x}_0) p_{r2}(\mathbf{y}_t | \mathbf{x}_0) p_f(\mathbf{y}_t | \mathbf{x}_0)}{Z_t} \xrightarrow{\text{Transition kernel}} \tilde{p}(\mathbf{y}_t | \mathbf{y}_s)$$

where  $p_{r1}(\mathbf{y}_t | \mathbf{x}_0)$  is the marginal distribution defined by SDE conditioned on  $\mathbf{x}_0$ ,

$$p_{r2}(\mathbf{y}_t | \mathbf{x}_0) \propto \exp(-\lambda_s \mathcal{E}_s(\mathbf{y}_t, \mathbf{x}_0, t)), p_f(\mathbf{y}_t | \mathbf{x}_0) \propto \exp(-\lambda_i \mathcal{E}_i(\mathbf{y}_t, \mathbf{x}_0, t)).$$

Equivalent

EGSDE

Euler-Maruyama  
solver

$$d\mathbf{y} = [\mathbf{f}(\mathbf{y}, t) - g(t)^2 (s(\mathbf{y}, t) - \lambda_s \nabla_{\mathbf{y}} \mathcal{E}_s(\mathbf{y}, \mathbf{x}_0, t) - \lambda_i \nabla_{\mathbf{y}} \mathcal{E}_i(\mathbf{y}, \mathbf{x}_0, t))] dt + g(t) d\bar{\mathbf{w}} \xrightarrow{\text{solver}} p(\mathbf{y}_t | \mathbf{y}_s)$$

# EGSDE as Product of Experts

Zhao et al, NeurIPS 2022

$$\tilde{p}(\mathbf{y}_t | \mathbf{x}_0) = \frac{p_{r1}(\mathbf{y}_t | \mathbf{x}_0) p_{r2}(\mathbf{y}_t | \mathbf{x}_0) p_f(\mathbf{y}_t | \mathbf{x}_0)}{Z_t}$$

↓  $t = 0$

$$\tilde{p}(\mathbf{y}_0 | \mathbf{x}_0) = \frac{p_{r1}(\mathbf{y}_0 | \mathbf{x}_0) p_{r2}(\mathbf{y}_0 | \mathbf{x}_0) p_f(\mathbf{y}_0 | \mathbf{x}_0)}{Z_0}$$

↑

$$\textcircled{\mathbf{y}_M} \longrightarrow d\mathbf{y} = [\mathbf{f}(\mathbf{y}, t) - g(t)^2 (s(\mathbf{y}, t) - \lambda_s \nabla_{\mathbf{y}} \mathcal{E}_s(\mathbf{y}, \mathbf{x}_0, t) - \lambda_i \nabla_{\mathbf{y}} \mathcal{E}_i(\mathbf{y}, \mathbf{x}_0, t))] dt + g(t) d\bar{\mathbf{w}} \longrightarrow \textcircled{\mathbf{y}_0}$$

realistic expert 1   realistic expert 2   faithful expert



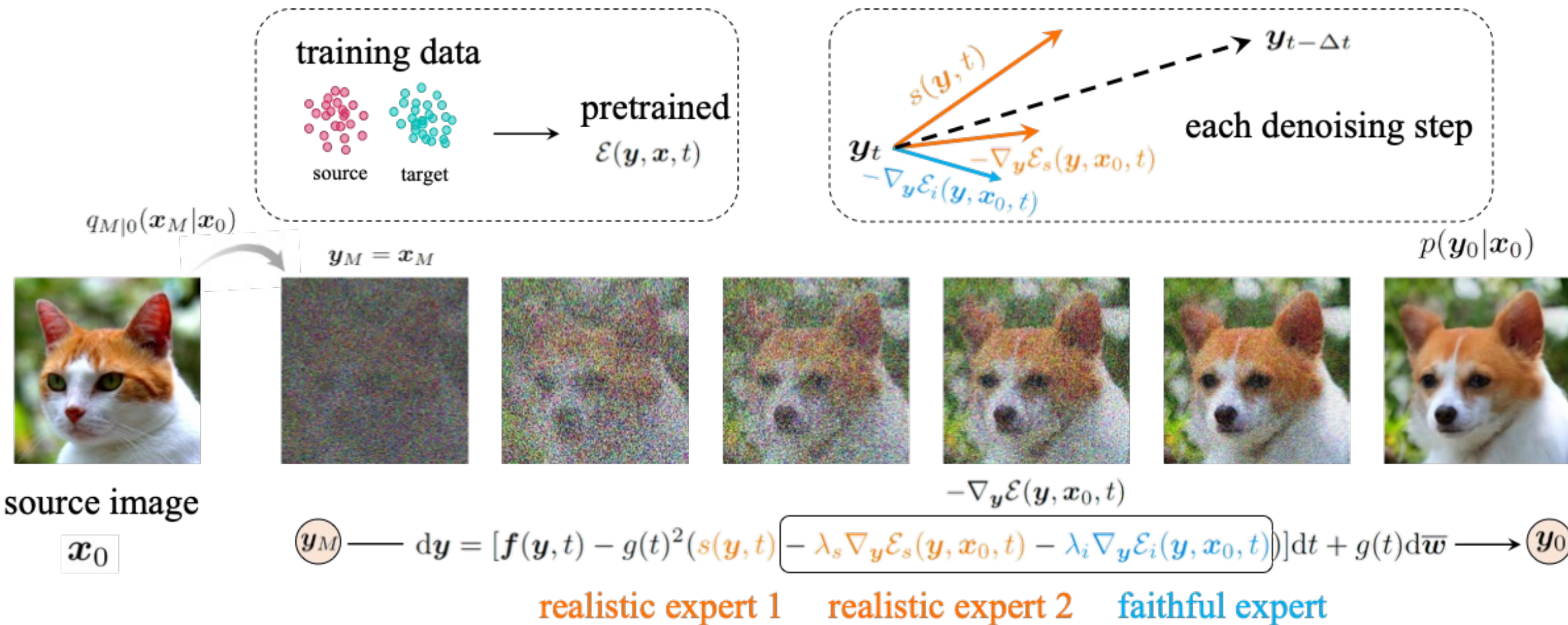
source image  
 $\mathbf{x}_0$



translated image

# Energy-Guided Stochastic Differential Equations ( EGSDE )

Zhao et al, NeurIPS 2022



# Results

Zhao et al, NeurIPS 2022

Model	<i>Realistic</i>	<i>Faithful</i>		<i>Human Evaluation, Both</i>	
	FID ↓	L2 ↓	PSNR ↑	SSIM ↑	AMT ↑
	Cat → Dog				
CycleGAN* [54]	85.9	-	-	-	-
MUNIT* [17]	104.4	-	-	-	-
DRIT* [25]	123.4	-	-	-	-
Distance* [3]	155.3	-	-	-	-
SelfDistance* [3]	144.4	-	-	-	-
GCGAN* [10]	96.6	-	-	-	-
LSeSim* [52]	72.8	-	-	-	-
ITTR (CUT)* [53]	68.6	-	-	-	-
StarGAN v2 [8]	54.88 ± 1.01	133.65 ± 1.54	10.63 ± 0.10	0.27 ± 0.003	-
CUT* [34]	76.21	59.78	17.48	<b>0.601</b>	79.6%
ILVR [7]	74.37 ± 1.55	56.95 ± 0.14	17.77 ± 0.02	0.363 ± 0.001	75.4%
SDEdit [30]	74.17 ± 1.01	47.88 ± 0.06	19.19 ± 0.01	<b>0.423 ± 0.001</b>	65.2%
EGSDE	<b>65.82 ± 0.77</b>	<b>47.22 ± 0.08</b>	<b>19.31 ± 0.02</b>	0.415 ± 0.001	-
EGSDE <sup>†</sup>	<b>51.04 ± 0.37</b>	62.06 ± 0.10	17.17 ± 0.02	0.361 ± 0.001	-

EGSDE :  $\lambda_s = 500, \lambda_i = 2, M = 0.5T$

EGSDE<sup>†</sup>:  $\lambda_s = 700, \lambda_i = 0.5, M = 0.6T$

# Results

Zhao et al, NeurIPS 2022

Model	FID ↓	L2 ↓	PSNR ↑	SSIM ↑	AMT ↑
Cat → Dog					
CycleGAN* [54]	85.9	-	-	-	-
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SelfDistance* [3]	144.4	-	-	-	-
EGSDE	86.6	-	-	-	-
StarGAN v2 [8]	54.88 ± 1.01	133.65 ± 1.54	10.63 ± 0.10	0.27 ± 0.003	-
CUT* [34]	76.21	59.78	17.48	<b>0.601</b>	79.6%
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EGSDE <sup>†</sup>	<b>51.04 ± 0.37</b>	62.06 ± 0.10	17.17 ± 0.02	0.361 ± 0.001	-

EGSDE outperforms the SBDMs-based methods in almost all metrics

EGSDE :  $\lambda_s = 500, \lambda_i = 2, M = 0.5T$

EGSDE<sup>†</sup>:  $\lambda_s = 700, \lambda_i = 0.5, M = 0.6T$



# Results

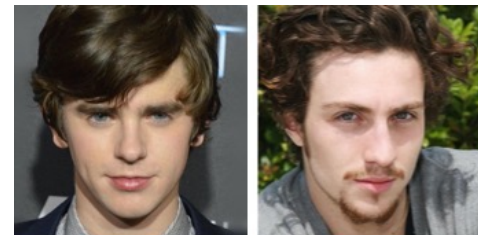
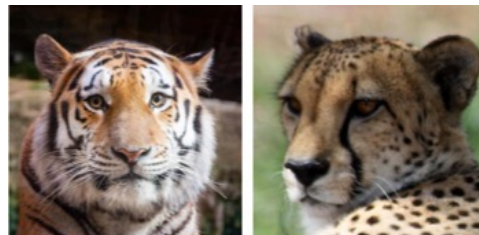
Zhao et al, NeurIPS 2022

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EGSDE <sup>†</sup>	<b>51.04 ± 0.37</b>	62.06 ± 0.10	17.17 ± 0.02	0.361 ± 0.001	-

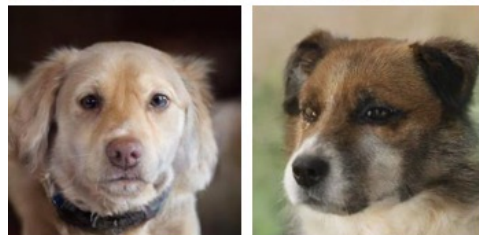
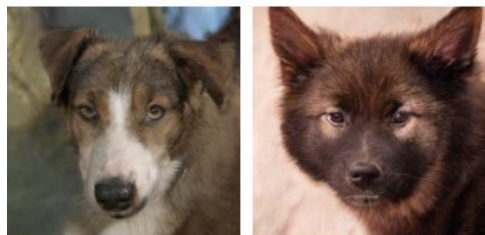
**EGSDE outperforms the current state-of-art GANs-based methods**

# Results

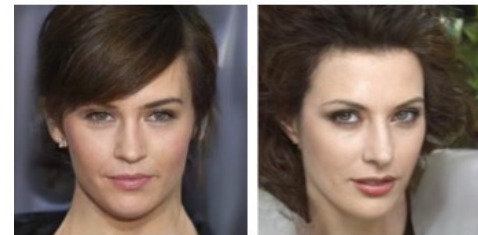
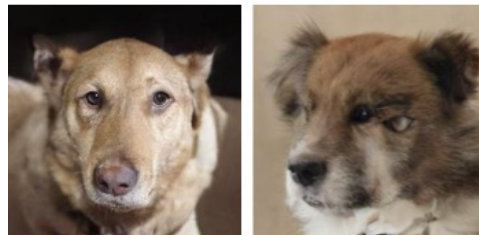
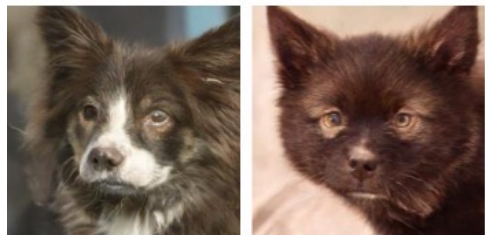
Source



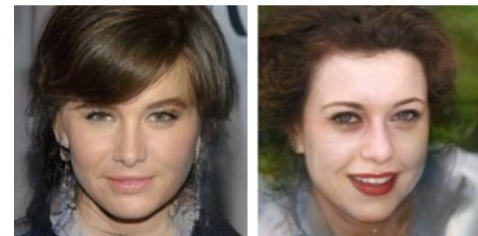
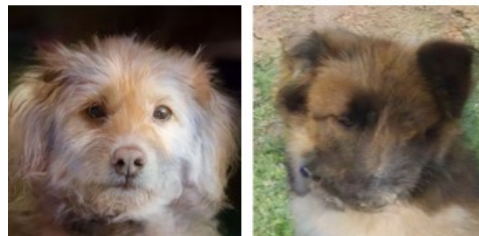
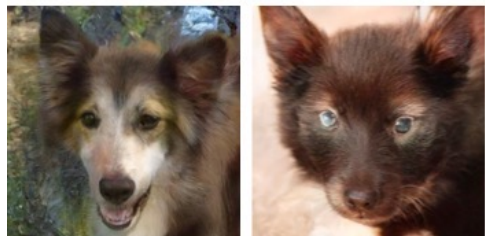
Ours



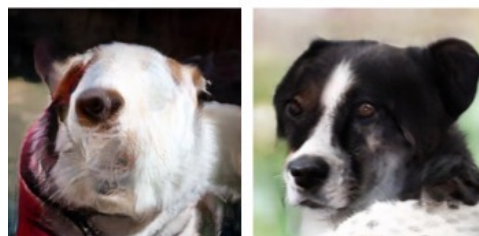
SDEdit



ILVR



CUT



Cat → Dog

Wild → Dog

Male → Female

# Results

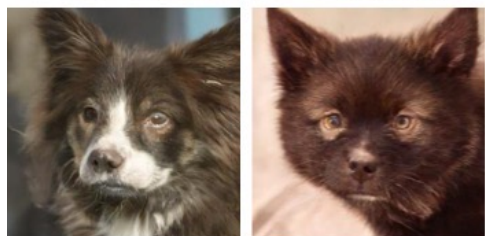
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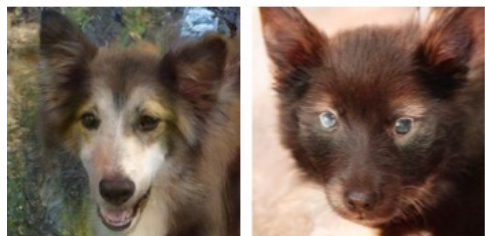
Ours



SDEdit



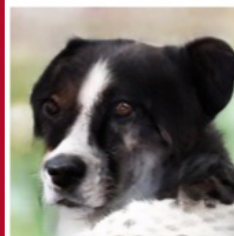
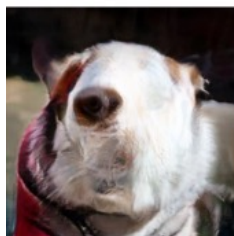
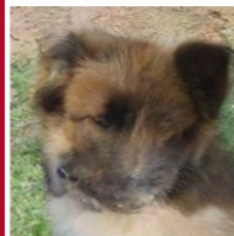
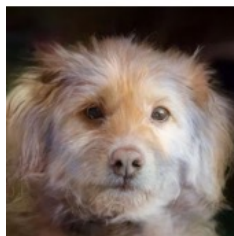
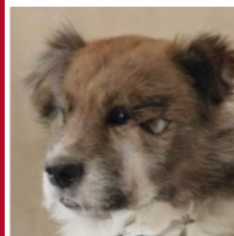
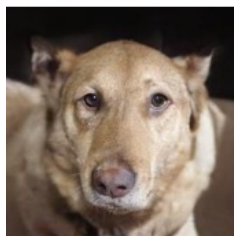
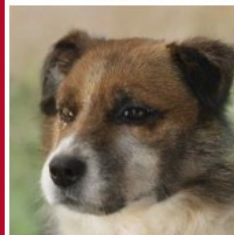
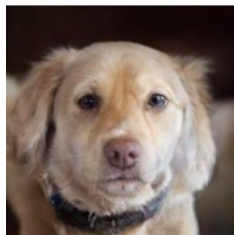
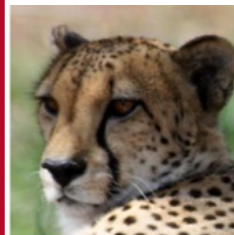
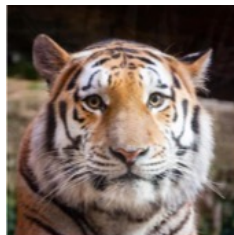
ILVR



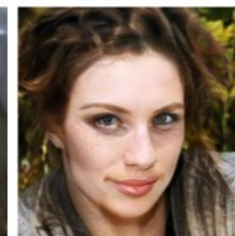
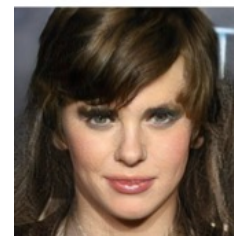
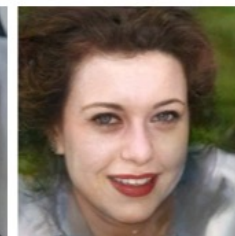
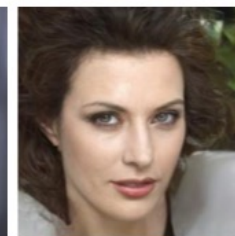
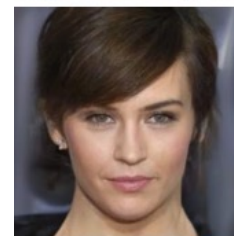
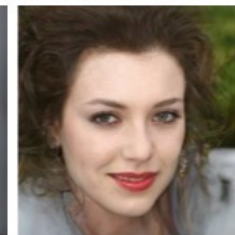
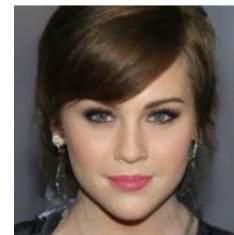
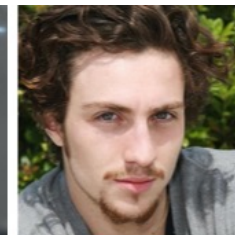
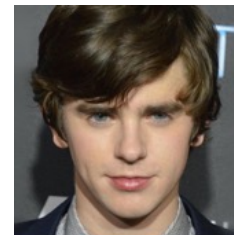
CUT



Cat → Dog

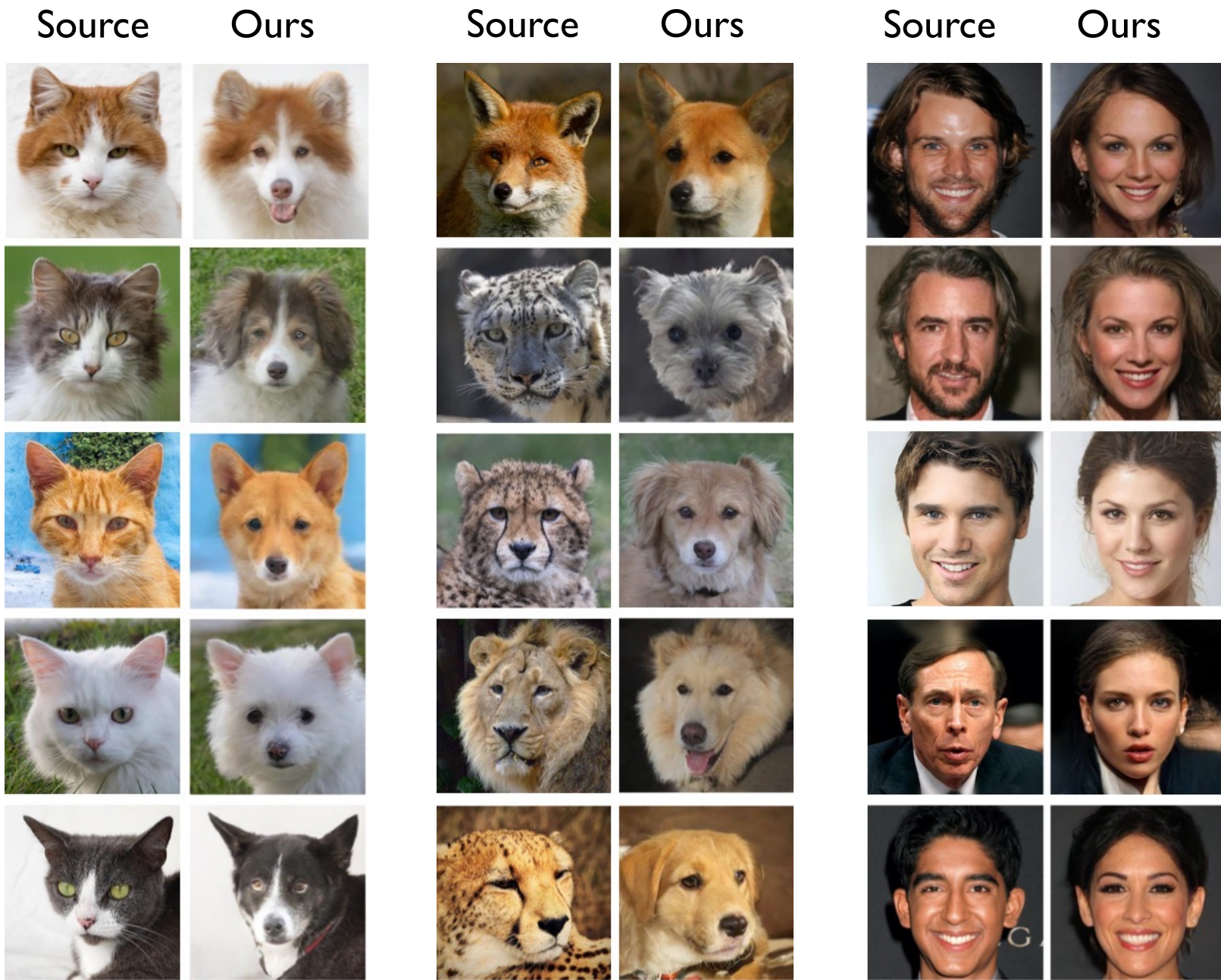


Wild → Dog



Male → Female

# Results



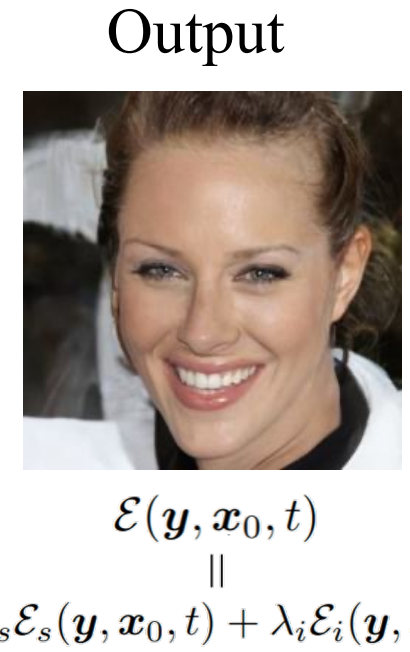
Cat → Dog

Wild → Dog

Male → Female

# The function of each expert

Zhao et al, NeurIPS 2022



## The choice of initial time M



Source

$M = 0.3T$

$M = 0.4T$

$M = 0.5T$

$M = 0.6T$

$M = 0.7T$

EGSDE in a big picture

# The Connection with Classifier Guidance

*Zhao et al, NeurIPS 2022*

EGSDE: a general framework that employs **an energy function** with **domain knowledge** to guide the inference process

$$d\mathbf{x} = [\mathbf{f}(\mathbf{x}, t) - g(t)^2(\mathbf{s}(\mathbf{x}, t) - \nabla_{\mathbf{x}}\mathcal{E}(\mathbf{x}, c, t))]dt + g(t)d\bar{\mathbf{w}}$$



# The Connection with Classifier Guidance

Zhao et al, NeurIPS 2022

EGSDE: a general framework that employs **an energy function** with **domain knowledge** to guide the inference process

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$$\mathcal{E}(\mathbf{x}, c, t) \propto -\lambda \log p_t(c|\mathbf{x})$$

Classifier Guidance

$$d\mathbf{x} = [\mathbf{f}(\mathbf{x}, t) - g(t)^2(\mathbf{s}(\mathbf{x}, t) + \lambda \nabla_{\mathbf{x}} \log p_t(c|\mathbf{x}))]dt + g(t)d\bar{\mathbf{w}}$$

# The Connection with Classifier Guidance

Zhao et al, NeurIPS 2022

EGSDE: a general framework that employs **an energy function** with **domain knowledge** to guide the inference process

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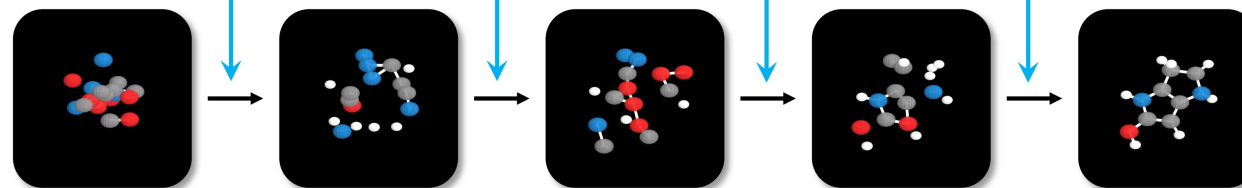
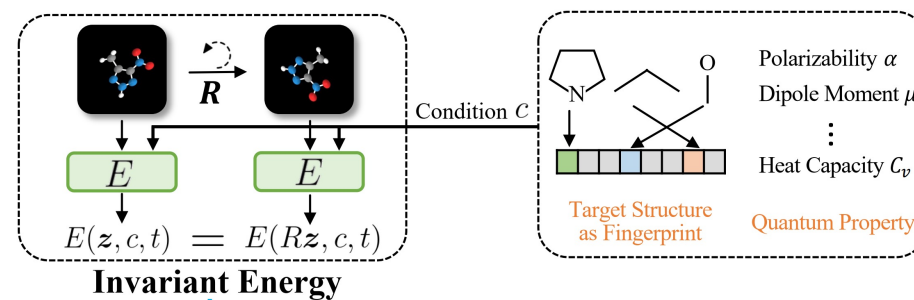
The classifier guidance can be regarded as a special design of energy function

# What's next?

Bao & Zhao et al., arxiv 2022

## Controllable 3D molecule generation

- Equivariant energy guidance
- Evaluated in inverse molecular design



$$z_T \leftarrow dz = [f(t)z + g(t)^2 \left( \frac{1}{\sqrt{\beta_{t|0}}} \epsilon_{\theta}(z, t) + \underbrace{(\nabla_x E(z, c, t) - \overline{\nabla_x E(z, c, t)}, \nabla_h E(z, c, t))}_{\text{Equivariant Energy Guidance}} \right)] dt + g(t) d(\tilde{w}_x, \tilde{w}_h) \rightarrow z_0$$

Target structure									
EDM									
EEGSDE (ours)									

Thank you!

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